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Theoretical Development of Biaxial Fabric Prestressed Composites under Tensile or Flexural Loading

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Abstract—In this study, we extend our development of the classical lamination theory of laminated composite with the presence of biaxial fabric prestressed. The aim of this paper is to describe the development of the fibre prestressed composite and its effect on the composite's internal stresses when subjected to tensile or flexural loading. The biaxial fabric prestress of the plain-weave composite could efficiently reduce the overall tensile stress within the composite lamina due to inducing compressive residual stress imparted from releasing the fibre pretension load. Thereby, the fibre prestressed composite could withstand more external tensile or flexural stress than non-prestressed counterparts did.

Keywords: Biaxial prestress; plain-weave fabric; residual stress; tensile and flexural stresses; development of classical lamination theory

1. Introduction

Currently, there is a need for high strength and lightweight materials in widespread applications such as in the aerospace, civil, automotive and sporting goods industries. For that reason, fibre-reinforced composite materials are now of great interest due to their high strength to weight ratio in comparison with most metals. As it is well known, the cost of composites is shared between its materials and its fabrication, for instance, the cost of fibreglass-reinforced polymer is approximately 60% materials and 40% fabrication [1]. Therefore, the focus on improving the fabrication techniques is still reasonable if they can enhance the mechanical behaviour of the composites. Unfortunately, residual stresses are generated within the composites during manufacturing process [2]. These stresses could considerably reduce the mechanical properties of the fibre reinforced composite. Residual stresses can develop in composite materials due to several reasons such as the chemical shrinkage of the polymer matrix, the different thermo-mechanical properties of the constitutions, moisture absorption, and fibre pretension [3]. Residual stresses within the matrix can arise due to the phase change of the resin from liquid to solid state (chemical shrinkage). The mismatch in the coefficient of thermal expansion between the fibre and the matrix will produce residual stresses in the composite when it is cooled from its curing temperature [3,4]. Several methods have been used to minimise the detrimental effects of the induced residual stresses within the fibre reinforced composite such as optimisation of dwell cure cycle [5], curing the composite at low temperatures [6], using electron beam curing (gamma irradiation) [7], using expanding monomers [8], inserting shape memory alloy wires [9], and using fibre pretension (prestress) method [10].



Fibre pretension could be one of the available options for improving the mechanical properties of polymeric matrix composites (PMCs) without increasing their section dimensions or mass [11]. The pretension was applied to the fibre prior and during the matrix cure. Once the matrix was cured well, the fibre pretension has been released. Up to now, the improvement obtained in the mechanical properties of fibre-reinforced composites due to using the fibre-prestressing method is not well established. However, our early previous studies have effectively confirmed the advantages of fibre prestressed composites for plain-weave composites [12–16]. Fibre pretension (prestressing) during matrix cure has generally a positive effect on the composite mechanical properties as it induces compressive residual stresses in the matrix. According to the authors' opinion, the effect of fibre pretension on the composite's internal stress subjected to a tensile or a flexural external load was not considered yet.

Therefore, the main aim of this research is to investigate the effect of biaxial fibre prestress on the mechanical behaviour of the plain weave composite subjected to tensile or flexural loadings. Theoretical equations of the macro-mechanics lamination theory of the composite are developed to include the fibre prestressing effect.

2. Materials and method

The composite material used in this study consisted of E-glass plain weave fabric/polyester resin system. The mechanical properties of the composite's constituents are listed in Table (1). These properties were measured at a temperature of 25 °C.

Table (1): Mechanical properties of the composite's constituent materials

Property	E-glass plain-weave fabric	Polyester resin
Elastic modulus (E), GPa	70	2.77
Tensile strength, MPa	2200	61
Poisson's ratio (ν)	0.23	0.25
Thermal expansion coefficient (α), 10^{-6} m/(m °C)	120	5.4
Warp density \times Fill density of the fabric (ends/m)	285 \times 245	-

The analysis of the fibre reinforced composite is based on the classical lamination theory. According to the classical lamination theory, a plane state of stress uses a macro-mechanical analysis using the subscription 1 as the direction of the warp fibres and the subscription 2 as the direction of fill fibres [15], the stress-strain ($\sigma - \epsilon$) matrix was expressed by:

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{Bmatrix} \quad (1)$$

where the reduced stiffnesses Q_{11} , Q_{22} , Q_{12} and Q_{66} were equal to:

$$Q_{11} = \frac{E_1}{1-\nu_{12}\nu_{21}}, Q_{22} = \frac{E_2}{1-\nu_{12}\nu_{21}}, Q_{12} = \frac{\nu_{21}E_1}{1-\nu_{12}\nu_{21}} = \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}}, Q_{66} = G_{12} \quad (2)$$

where

G_{12} : Shear modulus of a composite.

E_1 : Elastic modulus of a composite along the direction of fibres

E_2 : Elastic modulus of a composite transverse to the direction of fibres

ν_{12} : The strain in the transversedirection over the strain in the direction of fibre when the stress is applied along the fibre's direction

ν_{21} : The strain in the direction of fibre over the strain in the transverse direction when the stress is applied along the transverse direction

The developed residual stress–strain relation of a laminated composite is due to different sources that could be rewritten in the following form:

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix}_{res} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix}_{res} \tag{3}$$

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix}_{res} = \begin{Bmatrix} \varepsilon_1^{pre} - \varepsilon_1^{ther} \\ \varepsilon_2^{pre} - \varepsilon_2^{ther} \\ 0 \end{Bmatrix} \tag{4}$$

If the composite was cured at a temperature different from the room temperature, thermal strain was induced in the final product. Thermal strain would generate tensile residual stresses within the composite due to the different thermal expansion properties of the composite’s constituents. Residual thermal strain was calculated as:

$$(\varepsilon_{1,2}^{ther})_{res} = \alpha_{1,2} \Delta T \tag{5}$$

where

$\alpha_{1,2}$: Coefficients of thermal deformation in the 1 and 2 directions, respectively

ΔT : Difference between the cure and cool-down temperatures

For the case of a plain–weave fabric pretensioned in the biaxial directions (i.e. the warp and the fill yarns), the induced strain ε_1 and ε_2 due to releasing the fibre pretension load after the matrix has cured well are equal to:

$$(\varepsilon_1^{pre})_{res} = \frac{(\sigma^{pre}V_f)_{1-dir}}{\bar{E}_1} - \frac{\bar{\nu}_{12}(\sigma^{pre}V_f)_{2-dir}}{\bar{E}_1} \tag{6}$$

$$(\varepsilon_2^{pre})_{res} = \frac{(\sigma^{pre}V_f)_{2-dir}}{\bar{E}_2} - \frac{\bar{\nu}_{21}(\sigma^{pre}V_f)_{1-dir}}{\bar{E}_2} \tag{7}$$

where

\bar{E}_1, \bar{E}_2 : Effective elastic modulus in warp and fill directions, respectively

$\bar{\nu}_{12}, \bar{\nu}_{21}$: Effective Poisson’s ratio

V_f : Fibre volume fraction

At the micro–mechanical level, the strain in both the fibre and the matrix were equal to the lamina strain due to strain compatibility[15]. Residual stresses in the fibre and the matrix at a ply level could be estimated as follows:

$$\sigma_f^{res} = E_f (\varepsilon_{1,2}^{pre} - \varepsilon_{1,2}^{ther})_{res} + \sigma^{pre} \tag{8}$$

$$\sigma_m^{res} = E_m (\varepsilon_{1,2}^{pre} - \varepsilon_{1,2}^{ther})_{res} \tag{9}$$

The equivalent elastic properties according to Gay’s approach were equal to[15]:

$$\left. \begin{aligned} \bar{E}_1 &\approx \beta E_1 + (1 - \beta)E_2 \\ \bar{E}_2 &\approx (1 - \beta)E_1 + \beta E_2 \\ \bar{G}_{12} &= G_{12} \\ \bar{\nu}_{12} &\approx \frac{\nu_{12}}{(\beta + (1 - \beta)\frac{E_1}{E_2})} \\ \bar{\nu}_{21} &\approx \bar{\nu}_{12} \frac{E_2}{E_1} \end{aligned} \right\} \tag{10}$$

where

$$\beta = \frac{n_1}{n_1+n_2} \tag{11}$$

Here n_1 and n_2 denote the number of yarns per meter along the warp and fill directions that are equal to 285 and 245, respectively.

The effective composite thermoelastic properties of a unidirectional composite according to the concentric cylinder approach, assuming fibre behave transversely isotropic properties, were equal to [15]:

$$\left. \begin{aligned} E_1 &= E_f V_f + E_m V_m + \frac{4(v_f - v_m)^2 V_f V_m}{V_m/k^F + V_f/k^M + 1/G_m} \\ E_2 &= \frac{2(1 - v_f V_f - v_m V_m) k E_1}{(E_1 + 4k v_{12}^2)} \\ G_{12} &= G_m \left(\frac{G_m V_m + G_f (1 + V_f)}{G_f V_m + G_m (1 + V_f)} \right) \\ G_{m,f} &= \frac{E_{m,f}}{2(1 + v_{m,f})} \end{aligned} \right\} \tag{12a}$$

and

$$\left. \begin{aligned} \nu_{12} &= v_f V_f + v_m V_m + \frac{(v_f - v_m)(1/k^M - 1/k^F) V_m V_f}{V_m/k^F + V_f/k^M + 1/G_m} \\ \nu_{21} &= \nu_{12} \left(\frac{E_2}{E_1} \right) \\ \alpha_1 &= \frac{\alpha_f E_f V_f + \alpha_m E_m V_m}{E_f V_f + E_m V_m} \\ \alpha_2 &= \alpha_m V_m + \alpha_f V_f + \left(\frac{v_f E_m - v_m E_f}{\frac{E_m}{V_f} + \frac{E_f}{V_m}} \right) (\alpha_f - \alpha_m) \end{aligned} \right\} \tag{12b}$$

The axial residual stress that induced in the lamina due to the biaxial fabric prestress and thermal residual stresses is obtained from equation (3):

$$\sigma_1^{res} = Q_{11} \varepsilon_1^{res} + Q_{12} \varepsilon_2^{res} \tag{13}$$

where Q_{11} and Q_{12} in the above equation were determined from equation (2) that related the elastic properties of the composite's constituents with the fibre orientation angle; however, the ε_1^{res} and ε_2^{res} are the residual strains obtained from equations (3 and 4). Now, if the composite lamina is subjected to external axial tensile stress (σ_{axi}) in the direction of specimen length, the total axial stress (σ_{axi}^{total}) within the composite lamina at that direction would become:

$$\sigma_{axi}^{total} = \sigma_{axi} + \sigma_1^{res} \tag{14}$$

The axial tensile stress in the composite resulted from applying external axial load without considering residual stresses is equal to:

$$\sigma_{axi} = \frac{P_{axi}}{A_{total}} \tag{15}$$

where σ_{axi} , P_{axi} and A_{total} represent the axial stress, axial load and the cross sectional area of the composite material, respectively. Consequently, by substituting equation (15) into equation (14), one can find the total internal axial stress in the prestressed composite:

$$\sigma_{axi}^{total} = \frac{P_{axi}}{A_{total}} + \sigma_1^{res} \tag{16}$$

In the case of the prestressed lamina subjected to flexural loading, the total internal flexural stress in the lamina is equal to:

$$\sigma_{flex}^{total} = \mp \sigma_{flex} + \sigma_1^{res} \tag{17}$$

The \mp sign represents the tensioned and compressed regions located at the lower and upper sides of the neutral axis of the beam under flexural loading. σ_{flex} is the applied flexural stress in the composite lamina.

3. Results and discussion

The tensile residual stresses in the fabric when it was prestressed with different levels and released after matrix cure are listed in Table (2). It is very clear that increasing the biaxial fibre prestressing level could increase both the tensile residual stress in the fibre and the compressive residual stress within the matrix. On the other hand, the induced residual stresses within the matrix at warp and fill directions of the fabric were not equal due to using unequal yarn number per meter along the principal directions of the used E-glass fabric.

Table (2): Tensile residual stresses in the fibre after matrix cure due to applying different biaxial fabric prestressing levels

Biaxial fabric prestressing (MPa)	Tensile residual stress in the fibre (MPa)	Residual stress in the matrix (warp direction) (MPa)	Residual stress in the matrix (fill direction) (MPa)
25	5.841	-1.14	-1.26
50	11.68	-2.27	-2.53
75	17.52	-3.39	-3.75
100	23.30	-4.52	-5.00

The axial stress in the lamina results from applying axial stress that was developed in equation (14) are shown in Figure (1). The fibre volume fraction of the composite lamina was equal to 11% with cross-sectional area of 3 mm (thickness) × 25 mm (width). The effect of the fibre prestressing was very clear as it could reduce the tensile stresses in the prestressed lamina when subjected to external axial tensile stress. The flexural stress development in the presence of the fibre prestress is shown in Figure (2). The results were obtained from using the developed flexural equation (equation 15). The total tensile flexural stress reduced with increasing the prestressing level; however, compressive flexural stress in the compressed side increased as the fibre prestress was increased.

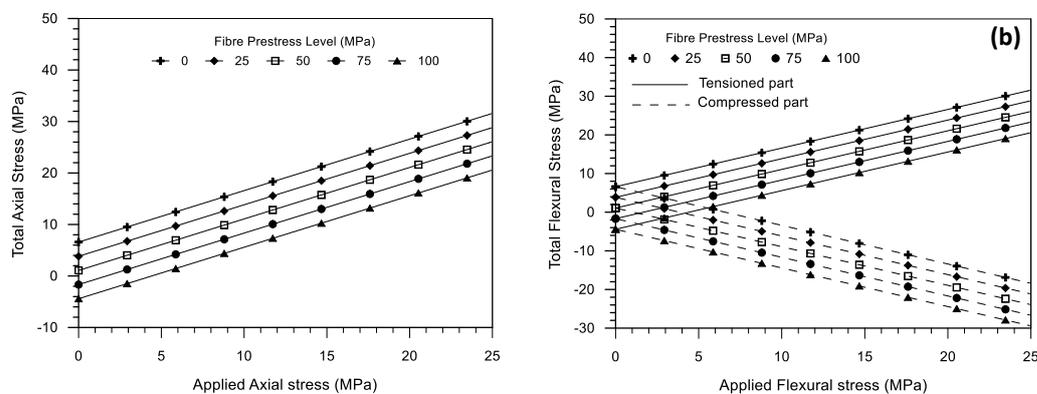


Figure (1): Total internal stress versus applied external stress in the composite lamina (E-glass/Polyester) cured at 50 °C and cooled down to 25 °C and prestressed with different levels. (a) Axial stress and (b) flexural stress.

In general, the tensile stress in the prestressed composite lamina could be reduced by increasing the biaxial fabric prestress [14,16]. This reduction is continued as the prestressing level has been increased, but this behaviour does not always lead to improvement on the mechanical properties of the composite as stated by many experimental studies[3]. The main reason behind this is attributed to the existence of interfacial shearing stress between the fibre and matrix [13], which is not considered in the classical lamination theory.

4. Conclusions

The biaxial fibre prestress of the composite reinforced with plain weave fabric could reduce the tensile residual stresses within the composite that were resulted from different thermo-mechanical properties of the composite's constitutions. The tensile and flexural stresses in the fibre prestressed laminated composite were reduced with increasing the level of the biaxial fibre prestress due to inducing compressive residual stresses within the lamina. This could improve the internal stress state of the composite subjected to external tensile or flexural loading.

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